IMPLEMENTATION OF THE ROSSELAND AND THE *P*1 RADIATION MODELS IN THE SYSTEM OF NAVIER-STOKES EQUATIONS WITH THE BOUNDARY ELEMENT METHOD

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ABSTRACT

The objective of this article is to develop a boundary element numerical model to solve coupled problems involving heat energy diffusion, convection and radiation in a participating medium. In this study, the contributions from radiant energy transfer are presented using two approaches for optical thick fluids: the Rosseland diffusion approximation and the *P*1 approximation. The governing Navier–Stokes equations are written in the velocity–vorticity formulation for the kinematics and kinetics of the fluid motion. The approximate numerical solution algorithm is based on a boundary element numerical model in its macro-element formulation. Validity of the proposed implementation is tested on a one-dimensional test case using a grey participating medium at radiative equilibrium between two isothermal black surfaces.

Keywords: compressible fluid flow, radiation models, boundary element method

1 INTRODUCTION

The Navier–Stokes equations set is commonly used as a frame for the solution of transport phenomena in a fluid flow. It provides a mathematical model of physical conservation laws of mass, momentum and energy considering specific rheological models describing non-convective fluxes of momentum and energy. In general, all three physically different mechanisms of heat transport can occur, that is diffusion, convection and radiation. The energy radiation phenomenon, which is a complex non-linear mode of heat transfer, gains importance at sufficiently high temperature [1]. At temperatures which are high enough these processes are essentially interdependent; energy transfer by one mechanism can influence heat exchange by the other mechanism and vice versa. The objective of this article is to develop a boundary element numerical simulation model to solve coupled problems involving heat energy diffusion, convection and radiation in a participating viscous compressible fluid flow.

The governing equation for radiative heat transfer is the radiative transfer equation [2], which is based on an energy balance for radiation passing through a differential volume in a participating medium in local thermo-dynamic equilibrium.

The radiation impact on overall heat transfer is conveyed in the energy equation, where, in the non-convective energy flux, besides the diffusion heat flux the radiative heat flux also needs to be taken into consideration. This is done in such a way that we include the term for the divergence of the radiative flux vector into the energy equation as the radiative energy source [3, 4]. The radiative transfer equations (RTE) is an integro-differential equation presenting a serious issue in computational fluid dynamics. Applying the chosen radiation model means, under the given physical circumstances, a simplification of the radiative transfer equation. In this study, the contributions from radiant energy transfer are presented using two approaches for optical thick fluids, that is the Rosseland diffusion approximation and the P1 approximation.

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2 GOVERNING EQUATIONS

The analytical description of the motion of a continuous viscous compressible heat radiation semi-transparent fluid is based on the conservation of mass, momentum and heat energy with associated rheological models for the non-convective fluxes of the momentum and heat energy and equations of state. The present development is focused on the laminar flow of compressible isotropic radiation semi-transparent fluid in solution domain $R = \Omega \times T$, where Ω stands for the two-dimensional plane domain bounded by boundary Γ defined by the outward-pointing unit normal $\vec{n}$, whilst T represents the time dimension of the transport phenomenon.

2.1 Conservation equations

The field functions of interest are the velocity vector field $\upsilon_i(r_j, t)$, scalar pressure field $p(r_j, t)$, temperature field $T(r_j, t)$ and the field of mass density $\rho(r_j,t)$, so that the mass, momentum and energy equations are given by the following set of non-linear equations:

$$\frac{\partial \upsilon_j}{\partial x_j} = -\frac{1}{\rho}\frac{D\rho}{Dt} = D, \tag{1}$$

$$\rho \frac{D\upsilon_i}{Dt} = -\frac{\partial \tau_{ij}}{\partial x_j} - \frac{\partial p}{\partial x_i} + \rho g_i \tag{2}$$

$$c\frac{DT}{Dt} = -\frac{\partial q_j^D}{\partial x_j} - \frac{\partial q_j^R}{\partial x_j} \tag{3}$$

in the Cartesian frame x_i, where ρ and c denote changeable mass density and isobaric specific heat capacity per unit volume, $c = c_p \rho$, t is the time, g_i is the gravitational acceleration vector and τ_{ij} represents the tensor components of the momentum diffusion, whilst the vector variables q_j^D and q_j^R are heat diffusion and radiation fluxes, respectively. The differential operator $D(\cdot)/Dt$ stands for the Stokes material derivative.

2.2 Rheological models for non-convective fluxes

The conservation eqns (2) and (3) contain two molecular diffusive fluxes, that is τ_{ij} and q_j^D, representing the diffusion of linear momentum and heat energy, respectively. The Newton linear momentum diffusion constitutive model for compressible viscous shear fluid is considered, such as

$$\tau_{ij} = 2\eta\dot{\varepsilon}_{ij} - \frac{2}{3}\eta D\delta_{ij}, \tag{4}$$

where $D = div\vec{\upsilon} = \dot{\epsilon}_{ii}$ represents the divergence of the velocity field or local expansion field, and η is a dynamic viscosity. For most heat transfer problems of practical importance, the simplification known as the Fourier law of heat diffusion is accurate enough, namely

$$q_i^D = -k^D \frac{\partial T}{\partial x_i}, \tag{5}$$

where k^D is the thermal heat conductivity.

All bodies at absolute temperature T emit electromagnetic radiation continuously over a wide range of wavelengths. At temperatures which are high enough, the simulation of the heat transfer becomes very complex. The mechanism of the heat transfer plays an important role in radiation which can present a great deal of the total heat flux. The governing equation for radiative heat transfer is the radiative transfer equation (RTE) [2], which is based on an energy balance for radiation passing through a differential volume in a participating medium in local thermodynamic equilibrium (LTE). The change in spectral intensity $i_\lambda(\vec{r})$ along a path from r to $r+dr$, where the time dependence of the intensity is neglected, expresses the quasi-steady form of the RTE

$$\frac{di_\lambda}{dl} = \vec{\nabla} i_\lambda \cdot \vec{l} = a_\lambda i_{\lambda b} - K_\lambda i_\lambda + \frac{\sigma_{S\lambda}}{4\pi} \int_{\omega=0}^{4\pi} i_\lambda \phi_\lambda d\omega \tag{6}$$

The spectral extinction coefficient $K_\lambda(\vec{r}) = a_\lambda(\vec{r}) + \sigma_{S\lambda}(\vec{r})$ is defined as the sum of the spectral absorption coefficient a_λ and the spectral scattering coefficient $\sigma_{S\lambda}$, $i_{\lambda b}$ is the black body isotropic spectral intensity and ϕ_λ is the scattering phase function. The eqn (6) is a first-order integro-differential equation for i_λ in a fixed direction $\vec{r}$. Due to the dependence on three spatial coordinates, two local direction coordinates and wavelength, an analytical solution is almost impossible for most engineering applications.

Thus, the eqn (6) has to be solved numerically using radiation transport models for spatial and directional dependencies and spectral models for the spectral dependency. In this study, we present an analysis of two common approximations for modelling radiative heat transfer that occurs in optically thick fluids. The contributions from the radiative energy transfer are presented using two approaches: the Rosseland diffusion approximation and the $P1$ approximation.

2.2.1 Rosseland diffusion approximation model

In diffusion approximation, we consider an absorbing and emitting medium with isotropic scattering ($\phi_\lambda = 1$). In an optically thick medium, radiation travels only a short distance before being absorbed or scattered. The Rosseland approximation reveals that the local spectral intensity i_λ depends only on the magnitude and the gradient of the local black body spectral intensity, $i_{\lambda b}(T)$, at that position [2]. The radiative flux vector for a grey medium can be approximated as

$$q_i^R(r_j) = -\frac{4\pi}{3K_R} \frac{\partial i_b(T)}{\partial x_i} = -\frac{16n^2\sigma T^3}{3K_R} \frac{\partial T}{\partial x_i} = -k^R \frac{\partial T}{\partial x_i} \tag{7}$$

where K_R is the Rosseland mean extinction coefficient, n is the refractive index and σ is the Stefan–Boltzmann constant. Although the Rosseland model provides a substantial simplification of the RTE and is recommended for use in problems where the optical differential thickness $\kappa_\lambda = \int_0^L K_\lambda dr$ exceeds 10 [2], it is often used for simulation of the radiation processes in many engineering applications. In analogy to eqn (5) the radiation heat conductivity k^R is introduced in eqn (7).

For the Rosseland radiation model it is possible to specify an adiabatic boundary condition, that is a zero-temperature gradient $dT / dn = 0$ at the solid wall or a specific wall temperature as the Dirichlet boundary condition. However, near a boundary, the diffusion approximation may not be accurate as the radiation is not isotropic [2]. To overcome this difficulty, the boundary condition at the edge of the medium is modified by using the effective jump boundary condition. Using the Deissler jump boundary condition concept for pure radiation [5], Goldstein and Howell introduced a similar concept for combined conduction and radiation [6]. In this model, the radiative heat flux at the wall boundary q_w^R is defined using the jump coefficient

$$\Psi_w = \frac{\sigma\left[T_w^4 - T^4\left(x \to 0\right)\right]}{q_w^R} \tag{8}$$

where T_w is the wall temperature and $T(x \to 0)$ is the extrapolated temperature of the medium at the wall. The jump coefficient is a function only of the conduction-radiation parameter $N_w = k^D K_R / 4\sigma T_w^3$, which expresses a measure of the ratio of the energy transferred by conduction and radiation. For large N_w the jump effect can be neglected, as heat conduction dominates over radiation effects near the wall. In general, the jump coefficient is approximated by a curve fit to the plot given in [3]

$$\Psi_w = \begin{cases} 1/2 & \text{for } \quad \mathrm{N}_w \le 0.01, \\ \dfrac{2x^3 + 3x^2 - 12x + 7}{54} & \text{for } 0.01 \le \mathrm{N}_w \le 10, \\ 0 & \text{for } \quad \mathrm{N}_w \ge 10, \end{cases} \tag{9}$$

where we note that $x = \log N_w$. To calculate a physical feasible jump temperature $T_w - T\left(x \to 0\right)$, a one-dimensional radiative energy balance for an infinitesimal medium layer adjacent to the solid wall is regarded, and delivers

$$q_w^R = -k^R \left.\frac{\partial T}{\partial n}\right|_w = \frac{\sigma\left[T^4(x \to 0) - T_w^4\right]}{\Psi_w} \tag{10}$$

As shown in the derivation of Ψ_w, the conditions for which the diffusion model is valid lead to the temperature jump $T_w - T\left(x \to 0\right)$ being small, so the difference $T_w^4 - T^4\left(x \to 0\right) \approx 4T_w^3\left(T_w - T\left(x \to 0\right)\right)$ can be linearized. The jump temperature follows the relation

$$T_w = T(x \to 0) - \frac{4\Psi_w}{3K_R}\left.\frac{\partial T}{\partial n}\right|_w . \tag{11}$$

2.2.2 Spherical harmonics approximation-$P1$ radiation model

The $P1$ radiation model relies on reducing the integral terms of the RTE to differential terms via a finite set of moment equations [2]. To develop the general PN method, the intensity at each position $\vec{r}$ is expressed as an expansion in a series of orthogonal harmonics and the series is truncated after a finite number of N terms. The $P1$ radiation model is the simplest case if only four terms in the series are retained. The method is a generalization of the Milne–Eddington equations analysed in [7, 2]. In engineering radiative transfer problems, the P1 model should typically be used for spectral optical thickness $\kappa_\lambda > 1$ [3].

A medium with spectral extinction coefficient $K_\lambda(\vec{r})$ and isotropic scattering is considered. The spectral incident radiation at position $\vec{r}$ is defined as

$$G_\lambda(\vec{r}) = \int_0^{4\pi} i_\lambda(\vec{r},\vec{l})d\omega, \tag{12}$$

where the integration takes over all solid angles [2]. Note that the spectral incident radiation divided by the speed of light G_λ / c is the spectral radiative energy density at location $\vec{r}$ in the radiation field. The $P1$ radiation model yields two spatial differential governing equations, one for the gradient of the directionally averaged spectral intensity

$$\vec{q}_\lambda^R = -\frac{1}{3K_\lambda}\vec{\nabla}G_\lambda = -\frac{1}{3\left[a_\lambda + \sigma_{S\lambda}\right]}\vec{\nabla}G_\lambda = -\Gamma_\lambda\vec{\nabla}G_\lambda, \tag{13}$$

where the parameter $\Gamma_\lambda = 1/3K_\lambda$, and another for the divergence of net radiative heat flux density vector

$$\vec{\nabla}\cdot\vec{q}_\lambda^R = a_\lambda\left[4\pi i_{\lambda b} - G_\lambda\right]. \tag{14}$$

Equations (13) and (14) can be combined to yield a second-order elliptic PDE for the incident radiation.

$$\vec{\nabla}\cdot\vec{q}_\lambda^R = -\vec{\nabla}\cdot\left[\Gamma_\lambda\vec{\nabla}G_\lambda\right] = a_\lambda\left[4\pi i_{\lambda b} - G_\lambda\right]. \tag{15}$$

This equation is simply a statement that the net radiative heat flux out of any region occupied by the medium is the difference between that emitted and that absorbed in the volume under consideration.

In the grey medium with constant absorption and extinction coefficients eqn (15) is simplified to a non-linear inhomogeneous modified Helmholtz equation:

$$\frac{\partial^2 G}{\partial x_j \partial x_j} - \beta G + b = 0 \text{ with } \beta = 3aK \text{ and } b = 4\beta\sigma T^4, \tag{16}$$

whilst the divergence of the radiation flux vector in eqn (3) can be expressed as the local radiation source term S^R:

$$\frac{\partial q_j^R}{\partial x_j} = -a\left(G - 4\sigma T^4\right) = -S^R. \qquad (17).$$

If it is assumed that the walls are diffuse grey surfaces, the eqn (16) is solved using Marshak boundary condition [3]

$$-\Gamma\left.\frac{\partial G}{\partial n}\right|_w = \frac{\varepsilon_w}{2\left(2-\varepsilon_w\right)}\left(4\sigma T_w^4 - G_w\right), \tag{18}$$

where $\in_w$ is the emissivity of the wall and the subscript w denotes the value of the indicated variable at the wall. Unlike the Rosseland diffusion approximation discussed above, there is no ambiguity about boundary conditions for the $P1$ approximation.

For the coupling of the radiative heat transport with the fluid dynamics, LTE is assumed and the time dependence of the radiative transfer equation is neglected. It follows from LTE that the temperature of the fluid and the corresponding radiative temperature in the medium are equal.

3 VELOCITY–VORTICITY FORMULATION OF NAVIER–STOKES EQUATIONS

In velocity–vorticity formulation, the fluid motion computation procedure may be partitioned into its kinetics and kinematics. The kinematics deals with the relationship and restriction between the velocity field at any given instant of time and the vorticity and local expansion field at the same instant, and is given by the following vector elliptic Poisson equation for the velocity vector

$$\frac{\partial^2 \upsilon_i}{\partial x_j \partial x_j} + e_{ijk} \frac{\partial \omega_k}{\partial x_j} - \frac{\partial D}{\partial x_i} = 0. \tag{19}$$

For the known vorticity and local expansion field functions, the corresponding velocity vector can be determined by solving eqn (19), provided that appropriate boundary conditions for the velocity are prescribed, that is the normal and tangential component of the velocity vector. The kinetic aspect of the fluid motion is governed by the vorticity transport equation.

$$\frac{\partial \omega_i}{\partial t} + \frac{\partial \upsilon_j \omega_i}{\partial x_j} = \nu_o \frac{\partial^2 \omega_i}{\partial x_j \partial x_j} + \frac{\partial \omega_j \upsilon_i}{\partial x_j} + \frac{1}{\rho_o} e_{ijk} \frac{\partial \rho g_k}{\partial x_j} + \frac{1}{\rho_o} e_{ijk} \frac{\partial f_k^m}{\partial x_j} \tag{20}$$

describing the redistribution of the vorticity in the fluid domain by different transport phenomena, for example diffusion, convection, twisting and stretching, whilst the buoyancy, compressibility, and the non-linear terms act as a source or strengthen terms. The vorticity transport equation is a highly non-linear partial differential equation due to the products of velocity and vorticity in convective and in stretching-twisting terms, and the velocity field function is kinematically dependent on vorticity and local expansion. However, strong coupling of the kinematics and kinetics can be clearly observed, even in the case of incompressible fluid.

The energy conservation equations are

$$c \frac{DT}{Dt} = \frac{\partial}{\partial x_j} \left(k_{eff} \frac{\partial T}{\partial x_j} \right) + S^R, \tag{21}$$

$$\frac{\partial^2 G}{\partial x_j \partial x_j} - \beta G + b = 0, \tag{22}$$

where $k_{eff} = k^D + k^R$ and $S^R = 0$ for the Rosseland radiation model and $k_{eff} = k^D$ and $S^R = -a\left(G - 4\sigma T^4\right)$ for the $P1$ one, respectively.

The pseudo body force term $\vec{f}^m$ and pseudo heat source term S_T^m were introduced into the vorticity fransport eqn (20) and into energy eqn (21) respectively, capturing the variable transport property effects, and given by expressions

$$\vec{f}^m = -\vec{\nabla} \times \left(\tilde{\eta}\vec{\omega}\right) + 2\vec{\nabla}\eta \times \vec{\omega} + 2\vec{\nabla}\vec{\upsilon} \cdot \vec{\nabla}\eta + \frac{4}{3}\vec{\nabla}\left(\eta D\right) - 2D\vec{\nabla}\eta - \tilde{\rho}\vec{a}, \tag{23}$$

while the pseudo heat source term is given by an expression

$$S_T^m = \vec{\nabla}\left(\tilde{k}\vec{\nabla}T\right) - \tilde{c}\frac{DT}{Dt}. \tag{24}$$

4 BOUNDARY-DOMAIN INTEGRAL EQUATIONS

In general, the set governing equations have to be transformed, using the Green identities or weighted residual techniques in combination with appropriate weighting function or fundamental solution, into boundary-domain integral equations. Boundary-domain integral formulation of the kinematics equation and the compressible vorticity equations has been presented by Skerget and Ravnik [8].

The integral representation of the non-linear heat energy diffusion convection transport equation is derived considering the linear parabolic diffusion differential operator yielding

$$L\left[T\right] + b = a_o \frac{\partial^2 T}{\partial x_j \partial x_j} - \frac{\partial T}{\partial t} + b = 0, \tag{25}$$

Therefore, the following integral representation can be evaluated

$$\begin{aligned} c\left(\xi\right)T\left(\xi, t_F\right) + a_0 \int_\Gamma \int_{t_{F-1}}^{t_F} Tq^* \, dtd\Gamma = {} & \frac{1}{c_0} \int_\Gamma \int_{t_{F-1}}^{t_F} \left(k\frac{\partial T}{\partial n} - c\upsilon_n T\right) u^* dtd\Gamma \\ & - \frac{1}{c_0} \int_\Omega \int_{t_{F-1}}^{t_F} \left(\tilde{k}\frac{\partial T}{\partial x_j} - c\upsilon_j T\right) q_j^* dtd\Omega \\ & + \frac{1}{c_0} \int_\Omega \int_{t_{F-1}}^{t_F} \left(T\upsilon_j \frac{\partial c}{\partial x_j} + cTD - \tilde{c}\frac{\partial T}{\partial t} - S^R\right) u^* dtd\Omega \\ & + \int_\Omega T_{F-1} u^*{}_{F-1} d\Omega. \end{aligned} \tag{26}$$

The boundary integrals describe the total heat flux on the boundary due to molecular diffusion and convection. The first domain integral gives the influence of the perturbated convection and the nonlinear diffusion flux, the second domain integral includes the non-linear material effects and radiation source, while the last domain integral is due to the initial temperature distribution effect on the development of the temperature field in subsequent time interval.

The incident radiation equation is an elliptic modified Helmholtz equation and, therefore, by employing the linear elliptic modified Helmholtz differential operator, we obtain the following expression

$$L[G] + b = \frac{\partial^2 G}{\partial x_j \partial x_j} - \beta G + b = 0 \tag{27}$$

and the corresponding boundary-domain integral representation to eqn (22) can be stated as

$$c(\xi)G(\xi)+\int_{\Gamma} Gq^{*}\,d\Gamma=\int_{\Gamma}\frac{\partial G}{\partial n}u^{*}\,d\Gamma+\int_{\Omega}4\beta\sigma T^{4}u^{*}\,d\Omega \tag{28}$$

where u^{*} is now the modified Helmholtz fundamental solution given by

$$u^{*}=\frac{1}{2\pi}K_{0}\left(\sqrt{\beta}r\right) \quad \text{and} \quad q^{*}=\frac{d_{n}}{2\pi r^{2}}\sqrt{\beta}rK_{1}\left(\sqrt{\beta}r\right), \tag{29}$$

whilst K_{α} is a modified Bessel function of the second kind of order α.

5 DISCRETIZED INTEGRAL EQUATIONS

The integral formulation of the governing PDE for the velocity, vorticity, temperature, pressure and incident radiation are written in a discretized form in which the integrals over the boundary and domain are approximated by a sum of the integrals over all boundary elements and over all internal cells.

Since the implicit set of equations is written simultaneously for all boundary and internal nodes, this procedure results in a fully populated influence and system matrices, resulting in large computing times and memory demands, which is especially true considering the fluid flow characterized by a high Reynolds number value. In order to improve the economics of the computation, we employ the macro-element approach [9]. The idea is to use a collocation scheme for integral equations for each domain cell separately and require that the field functions and their normal derivatives must obey the compatibility and equilibrium conditions over the domain cell boundaries. The final system of equations for the entire domain is then obtained by adding the sets of equations for each macro element, resulting in a sparse system matrix suitable to solve with iterative techniques [10, 11].

6 VALIDATION

To check the validity of the implemented Rosseland and the $P1$ radiation models we investigated a one-dimensional case. We consider a grey participating medium at radiative equilibrium between two isothermal black surfaces $(\in=1)$ at temperatures $T_h=600K$ and $T_c=300K$. We consider radiation as being coupled to the energy equation via Rosseland diffusion approximation with specified Dirichlet boundary condition and radiative heat transfer as a source term using the $P1$ radiation model with specified Marshak boundary condition.

The coefficient of the diffusion thermal heat conductivity is reduced to the value where $k^{D}=10^{-4}W/mK$. The test example is analysed for a grey medium with optical thickness $\kappa_L=aL=10$ and $\kappa_L=2$. The influence of the natural convection was neglected. The exact results of the RTE are available for these cases and can be found in [1]. Figures 1 and 2 show the non-dimensional temperature $T^{*}=\left(T^{4}-T_c^{4}\right)/\left(T_h^{4}-T_c^{4}\right)$ versus non-dimensional coordinate $x^{*}=x/L$ and compare the results of the $P1$ and the Rosseland radiation models with the results of [1].

The results reveal good agreement between present numerical results and the exact solution of Modest [1]. Results also reveal a temperature discontinuity (a sharp temperature profile) at the walls. In a limiting case of a transparent medium $\kappa_L\to 0$, the non-dimensional temperature takes the value of 0.5. The temperature slip at the walls decreases as the optical thickness increases and vanishes as $\kappa_L\to\infty$.

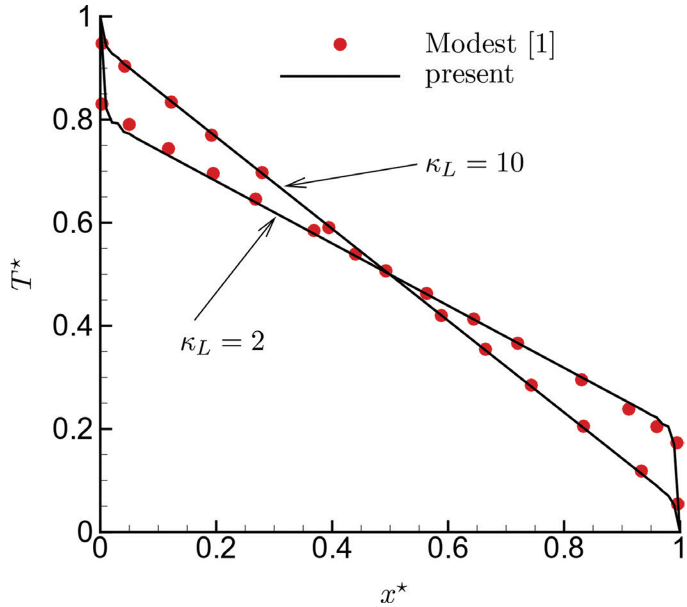

Figure 1: The comparison of pure radiation simulation results for the dimensionless temperature versus non-dimensional coordinate for a grey medium between two plates. Results of the P1 model are shown versus benchmark results of Modest [1].

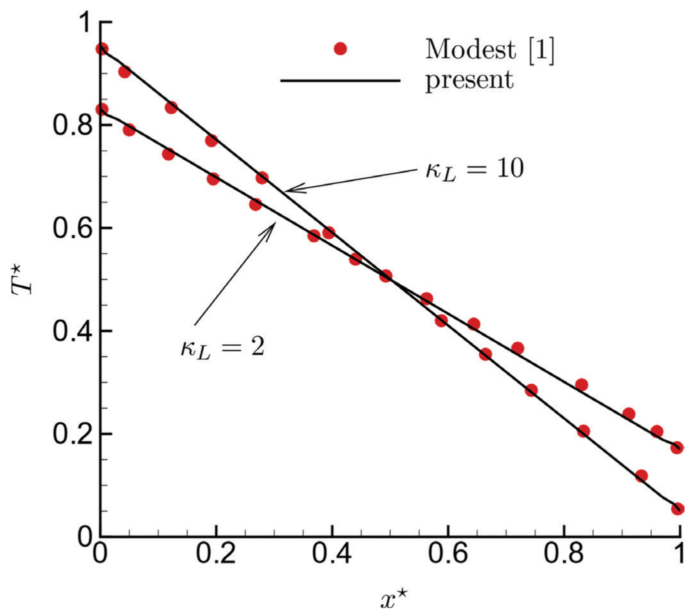

Figure 2: The comparison of pure radiation simulation results for the dimensionless temperature versus non-dimensional coordinate for a grey medium between two plates. Results of the Rosseland model are shown versus benchmark results of Modest [1].

While the Rosseland model reveals the temperature discontinuity at the walls, the $P1$ model, due to low heat conductivity, results in a more physically realistic temperature profile at the wall.

7 CONCLUSIONS

We have implemented two approaches to solve for radiation energy transport within an optically thick fluid. The choice of any one of the solution methods will depend upon the computational effort needed for the solution as well as for the accuracy of the solution. Based upon the results of this study, there are some clear recommendations. The Rosseland approximation is easy to implement and economical in computing needs. These issues are especially important with regard to incorporating internal radiant heat transfer into existing heat transfer codes. However, this simplicity carries with it a significant source of inaccuracy – the inability of this method to capture the physics of the thermal boundary layer near the walls. Indeed, this downfall may lead to significant errors, especially in systems such as this where the thermal boundary layers are important for driving flow. The $P1$ model is slightly more difficult to implement, since it requires solving an additional coupled partial differential equation for an additional field variable, the incident radiation. In addition, the computational effort needed to solve with the P1 model is slighter greater than that needed for the Rosseland diffusion methods.

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