NUMERICAL SIMULATION OF CONVECTIVE FLOW IN A NON-DARCY POROUS CAVITY FILLED WITH NANOFLUID

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ABSTRACT

Suspensions of nanoscale particles and fluids have been recently subject of intense research, since it was proved that they considerably improve heat transfer capabilities of the fluid which can be crucial in several technological processes. Several applications can be found in the field of porous media flow, such as oil recovery systems, thermal and geothermal energy, nuclear reactors cooling. Since nanofluids are a mixture of a solid and fluid phase, in general, the two phase mathematical model would be the most appropriate to use. However, due to very small size of nanoparticles (1–100 nm) can be assumed, that they behave as a water molecule and a single phase model along with empirical correlations for nanofluid properties can be used. In the present study a convective flow through porous cavity fully saturated with nanofluid is analyzed in detail using the single phase mathematical model based on the Navier-Stokes equations taking into account the non-Darcy parameters. The mathematical model is written at a macroscopic level enabling the simulation of the porous media flow. The solutions are obtained with the in house numerical code based on the Boundary Element Method, which was already proved to have some unique advantages when considering fluid flow problems in different configurations. The effects of the presence of different types of nanoparticles as well as the porous matrix were investigated in detail for different values of governing parameters in order to examine the improved heat transfer characteristics of added nanoparticles.

Keywords: boundary element method, Darcy-Brinkman-Forchheimer formulation, nanofluids, porous medium flow

1 INTRODUCTION

The phenomena of convection in a fluid saturated porous media has been a subject of intense research in last several decades due to many applications in contemporary technologies such as building insulation systems, geothermal engineering, pollutant transport in underground, nuclear energy systems, just to name a few. Effective cooling and heating techniques are important in many engineering applications and are very limited using conventional fluids, such as water, engine oil or ethylene glycol mixtures. To enhance heat transfer or cooling processes it is desirable to combine fluids with a material which proves high thermal conductivity. The combination of fluid with small particles (nanoparticles) of metal appreciably improves the heat transfer characteristics of fluid while other properties remain the same. The idea of nanofluids was first introduced by Choi and Estman [1]. Later, many theoretical and experimental studies have been published where suitable models for effective thermal conductivity as well as the viscosity of nanofluid were developed. A comprehensive review of existing studies was published by Kakac and Pramuanjaroenkij [2] and Haddad *et al*. [3].

The mathematical models for nanofluids are generally based on the single-phase or two-phase approach. The single phase approach predicts the thermal equilibrium of the fluid phase and solid particles. It is also assumed that the fluid and solid particles have same local velocities. This assumption is valid only if the solid particles are small enough ($< 1–100\ nm$) and at low concentration of nanoparticles (2.5%–5%). In that case the solid-liquid mixture can be considered as a single-phase fluid with modified properties e.g. density, specific heat,

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thermal conductivity. A single-phase nanofluid model was first developed by Tiwari and Das [4] and was recently used in porous media applications e.g. in, [5, 6]. On the other hand, the nanofluids are two-phase mixtures in general, and two-phase model is a more accurate choice to describe transport processes. This type of model was proposed by Buongiorno [7] and predicts the interaction of the fluid and solid particles, Brownian forces, Brownian diffusion, sedimentation and dispersion. Recently the model was used on some porous media applications in [8, 9].

In the present paper, the single-phase nanofluid mathematical model was used to analyze natural convection phenomena in porous enclosure. The extended porous media model with Dracy-Brinkman-Forchheimer momentum equation was used to simulate porous media flow. In order to efficiently solve the governing set of partial differential equations, the Boundary Element Method based numerical code was used as proposed in [10, 11]. Therefore the governing set of equations is transformed into a velocity-vorticity formulation. The numerical code was already used for several applications of pure fluid flow [12, 13], as well as porous media applications [14, 15]. Several numerical results are presented in order to analyze the influence of nanofluid in combination with porous media on heat transfer and fluid flow characteristics.

2 PROBLEM DEFINITION AND GOVERNING EQUATIONS

The natural convection phenomena of nanofluids were simulated on a problem of two-dimensional porous square cavity, where the vertical walls are maintained at different temperatures (Fig. 1). The corresponding boundary conditions for the problem are:

$$\begin{aligned} &v_x = v_y = 0, T = T_h \text{ at } x = 0, \quad 0 \le y \le 1, \\ &v_x = v_y = 0, T = T_c \text{ at } x = L, \quad 0 \le y \le 1, \\ &v_x = v_y = 0, \frac{\partial T}{\partial y} = 0 \text{ at } y = 0, \quad 0 \le x \le 1, \\ &v_x = v_y = 0, \frac{\partial T}{\partial y} = 0 \text{ at } y = L, \quad 0 \le x \le 1, \end{aligned} \tag{1}$$

where T is temperature, T_h and T_c are constant temperatures on the hot and cold wall respectively. Horizontal walls are assumed to be adiabatic $\partial T / \partial y = 0$ and non-slip boundary condition is prescribed on all walls.

Further assumptions are, that the solid matrix of porous medium is isotropic, homogenous and in thermal equilibrium with the fluid phase, which is a suspension of water and nanoparticles. In this study, *Cu* spherical nanoparticles were considered with thermophysical properties given in Table 1. The nanofluid properties are calculated with relationships to pure fluid and pure solid properties, and are linked with the solid volume fraction of nanoparticles, $\varphi = V_s/(V_s + V_f)$, where V_s and V_f are volume of solid particles and volume of fluid respectively. The nanofluid properties are calculated by the following expressions [3] (index f stands for the fluid phase and s for the solid phase):

- the effective density of nanofluid ρ_{nf}:

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s, \tag{2}$$

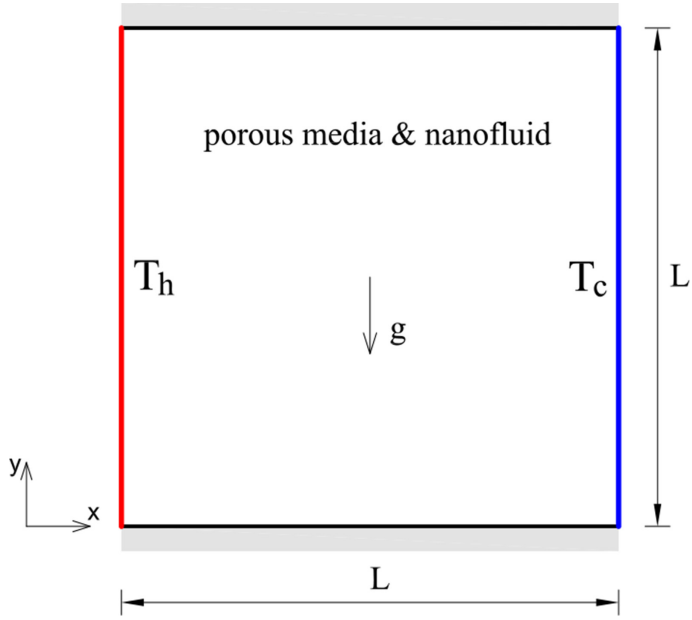

Figure 1: Computational domain with boundary conditions.

Table 1: Thermophysical properties of water and *Cu* solid nanoparticles [16].

	C_p [J/kgK]	ρ [kg/m^3]	k [W/mK]	β [$\times 10^{-5}K^{-1}$]	α [$\times 10^{-7}m^2/s$]
Water	4,179	997.1	0.613	21	1.47
Cu	385	8,933	400	1.67	1,163

- the effective dynamic viscosity μ_{nf}, Brinkman model [17]:

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}}, \tag{3}$$

- the heat capacitance $(c_p)_{nf}$:

$$(\rho c_p)_{nf} = (1-\varphi)(\rho c_p)_f + \varphi(\rho c_p)_s, \tag{4}$$

- the thermal expansion coefficient β_{nf}:

$$(\rho\beta)_{nf} = (1-\varphi)(\rho\beta)_f + \varphi(\rho\beta)_s, \tag{5}$$

- the effective thermal conductivity k_{nf}, Wasp model [18]:

$$k_{nf} = k_f \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + \varphi(k_f - k_s)}. \tag{6}$$

To describe the natural convection phenomena in porous media domain the mathematical model based on the conservation equations for mass, momentum and energy is used, suitable average over the representative elementary volume (REV) [19]. Macroscopic equations for nanofluid flow in porous media domain are:

- Continuity equation:

$$\vec{\nabla} \cdot \vec{v} = 0, \tag{7}$$

- Momentum equation:

$$\frac{1}{\phi}\frac{\partial \vec{v}}{\partial t} + \frac{1}{\phi^2}(\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{\rho_{nf}}\vec{\nabla}p - \beta_{nf}(T-T_0)\vec{g} + \frac{1}{\phi}\frac{\mu_{nf}}{\rho_{nf}}\nabla^2\vec{v} - \frac{1}{K}\frac{\mu_{nf}}{\rho_{nf}}\vec{v} - \frac{F\vec{v}\,|\vec{v}\,|}{K^{1/2}}, \tag{8}$$

- Energy equation:

$$\sigma\frac{\partial T}{\partial t} + (\vec{v}\cdot\vec{\nabla})T = \frac{k_e}{(\rho c_p)_{nf}}\nabla^2 T, \tag{9}$$

with following parameters: $\vec{v}$ volume averaged velocity vector, ϕ porosity, t time, p fluid pressure, T temperature, $\vec{g}$ gravitational acceleration, K permeability, F Forchheimer coefficient. In energy equation σ is the specific heat ratio $\sigma = \phi + (1-\phi)(\rho c_p)/(\rho c_p)_{nf}$, where $(\rho c_{p)\boldsymbol{p}}$ and $(\rho c_{\boldsymbol{p}})_{\boldsymbol{nf}}$ are heat capacitances of porous medium and nanofluid, respectively, and k_e is the effective conductivity of porous medium. In this study the thermal properties of solid matrix and the nanofluid are considered to be identical, resulting in $\sigma = 1$ and $k_e = k_{\boldsymbol{nf}}$ as this is common practice [6, 20].

A Brinkman-Forchheimer momentum equation is used with the additional Brinkman viscous term and the Forchheimer inertial term describing the nonlinear influences at higher velocities [21].

The equations are transformed into a non-dimensional form using the following dimensionless parameters:

$$\vec{v} \to \frac{\vec{v}}{v_0}, \vec{r} \to \frac{\vec{r}}{L}\, t \to \frac{v_0 t}{L}, \vec{g} \to \frac{\vec{g}}{g_0}, \quad p \to \frac{p}{p_0}, T \to \frac{(T-T_0)}{\Delta T}, \tag{10}$$

where $\vec{v}$ is volume averaged velocity vector, v_0 is a characteristic velocity given with an expression $v_0 = k_f/(\rho c_{\boldsymbol{p}})_f\, L$, k_f is fluid thermal conductivity, $(\rho c_{\boldsymbol{p}})_f$ is heat capacity for fluid phase and L is the characteristic length (e.g. length of one side of square cavity), $\vec{r}$ is. Moreover, T_0 is characteristic temperature $T_0 = (T_2 - T_1)/2$, where ΔT is characteristic temperature difference $\Delta T = T_2 - T_1$, p_0 is characteristic pressure $p_0 = 1bar$, while gravitational acceleration is $g_0 = 9.81$ m/s^2. Furthermore, the velocity-vorticity formulation is proposed by defining the vorticity vector as a curl of the velocity field $\vec{\omega} = \vec{\nabla} \times \vec{v}$ which separates the computational scheme into a kinematic and kinetic computational parts. The kinematic part is

governed with the velocity equation which is obtained from the mass conservation law (7) and reads as:

$$\nabla^2 \vec{v} + \vec{\nabla} \times \vec{\omega} = 0. \tag{11}$$

The kinetic part is governed by the energy and the vorticity transport equation, which is derived by taking the curl of the momentum eqn (8). The equations in the non-dimensional form are given as:

$$\begin{aligned}(\vec{v} \cdot \vec{\nabla})\vec{\omega} = (\vec{\omega} \cdot \vec{\nabla})\vec{v} - PrRa_T \phi^2 \frac{\beta_{nf}}{\beta_f} \vec{\nabla} \times T\vec{g} + Pr\phi \frac{\mu_{nf}}{\mu_f} \frac{\rho_f}{\rho_{nf}} \nabla^2 \vec{\omega} \\ - \frac{Pr}{Da} \phi^2 \frac{\mu_{nf}}{\mu_f} \frac{\rho_f}{\rho_{nf}} \vec{\omega} - \frac{F}{\sqrt{Da}} \phi^2 \mid \vec{v} \mid \vec{\omega},\end{aligned} \tag{12}$$

$$(\vec{v} \cdot \vec{\nabla})T = \frac{\alpha_{nf}}{\alpha_f} \nabla^2 T, \tag{13}$$

where α_{nf} is the thermal diffusivity of nanofluid $\alpha_{nf} = k_{nf}/(\rho c_p)_{nf}$ and α_f thermal diffusivity of pure fluid $\alpha_f = k_f/(\rho c_p)_f$. Since in this study only steady flow fields are considered, the time derivatives in the vorticity and energy equations, $\partial\vec{\omega}/\partial t$, $\partial T/\partial t$, are omitted.

The non-dimensional parameters in the governing equations are defined as: fluid Rayleigh number $Ra_T = g\beta_T \Delta T L^3 \rho_f (\rho c_p)/\mu_f k_f$, Prandtl number $Pr = \mu_f c_p/k_f$, Darcy number $Da = K/L^2$, porosity ϕ.

3 NUMERICAL METHOD

The algorithm, which solves the governing set of equations is based on the boundary element method, and solves the velocity-vorticity formulation of Navier-Stokes equations by a combination of single-domain and sub-domain BEM. All governing equations are written in an integral form which is obtained by using the Green's second identity for the unknown field function and for the fundamental solution of the Laplace equation [12, 13]. The integral form of all governing equations is given in [14] and is omitted in this paper.

At the beginning the boundary conditions of Dirichlet or Neumann type must be known. In this study, on all solid walls the no-slip boundary conditions and temperature or temperature flux are prescribed. The boundary conditions for vorticity are calculated as a part of the algorithm. The outline of the algorithm is as follows:

1. Calculation of nanofluid material properties using models (2)–(6).
2. Determination of porous media properties.
3. Calculation of unknown field functions:
   - Vorticity values on the boundary by single domain BEM from the kinematic eqn (11).
   - Velocity values in the domain by sub-domain BEM from the kinematic eqn (11).
   - Temperature values in the domain by sub-domain BEM from the energy eqn (13).
   - Vorticity values in the domain by sub-domain BEM from the vorticy transport eqn (12).
4. Convegence check; all steps are repeated until the required accuracy is achieved.

4 RESULTS AND DISCUSSION

The proposed numerical code was first validated by a comparison with the results published by Nguyen *et al*. [6], for a case of porous cavity saturated with the Cu-nanofluid. The results are presented in terms of average heat transfer given with the Nusselt number defined as an integral of the temperature flux through a wall: $Nu = k_{nf}/k_f \int_\Gamma \vec{\nabla} T \cdot \vec{n} d\Gamma$. The present and reference results are given in Table 2 for Prandtl number Pr = 6.2, solid volume fraction (φ = 0.0,0.025, 0.05) and other governing parameters (porous Rayleigh number Ra_p, Darcy number Da, and porosity ϕ). Good agreement between the result can be obtained. Furthermore, computations for several different governing parameters have been carried out in order to analyze the influence of different governing parameters on the fluid flow and heat transfer in porous cavity. Figure 2 represents isotherms for Cu-water nanofluid at porosity ϕ = 0.4 and

Table 2: Validation of the numerical code by a comparison of average Nu for a natural convection in porous media saturated with a nanofluid (Pr = 6.2) for different governing parameters.

				φ = 0.05			
		ϕ = 0.4		ϕ = 0.6		ϕ = 0.9	
Da	Ra_p	[6]	Present	[6]	Present	[6]	Present
10^{-2}	1,000	3.433	3.400	3.850	3.826	4.162	4.145
10^{-4}	1,000	9.117	9.132	9.590	9.743	9.901	10.154
10^{-6}	1,000	11.778	12.991	11.899	13.128	11.976	13.195
				ϕ = 0.4			
		φ = 0.0		φ = 0.025		φ = 0.05	
10^{-2}	10	1.007	1.008	1.081	1.083	1.160	1.162
10^{-2}	1,000	3.302	3.282	3.370	3.345	3.433	3.400
10^{-6}	1,000	11.867	13.238	11.847	13.131	11.778	12.991

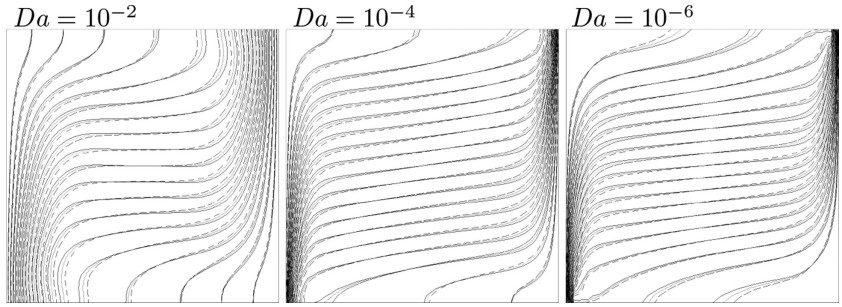

Figure 2: Temperature contour plots for ϕ = 0.4, Ra_p = 1,000 and various Da; solid lines φ = 0.0, dotted lines φ = 0.025, dashed lines φ = 0.05.

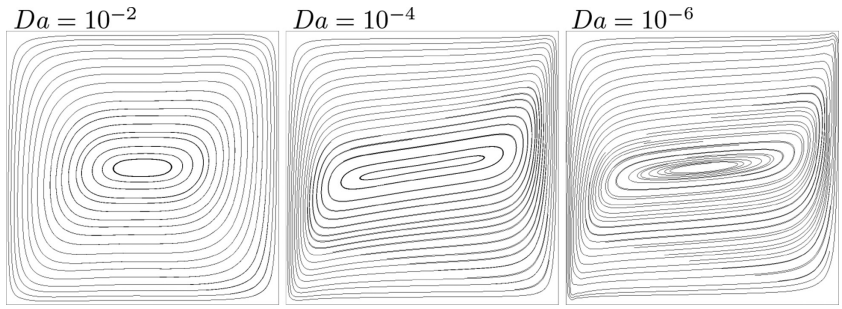

Figure 3: Streamlines for $\phi = 0.4$, $\varphi = 0.05$, $Ra_p = 1{,}000$ and various Da.

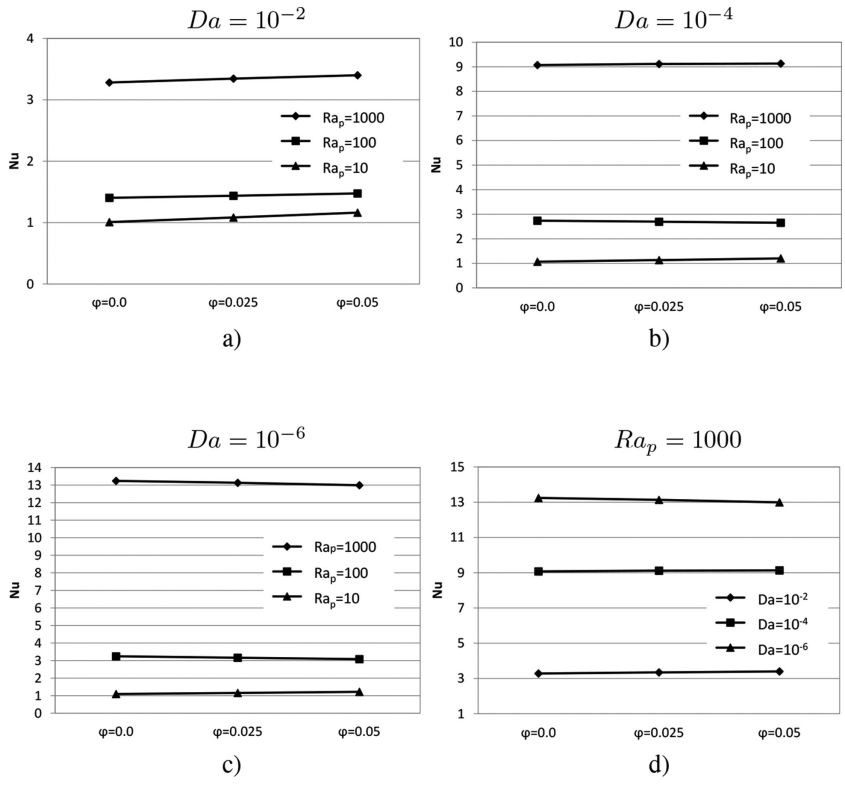

Figure 4: Average Nu for Cu-water nanofluid depending on solid volume fraction φ for different values of Ra_p and Da: (a) $Da = 10^{-2}$, (b) $Da = 10^{-4}$, (c) $Da = 10^{-6}$ and (d) $Ra_p = 1{,}000$; ($\phi = 0.4$).

$Ra_p = 1{,}000$ under different values of Da and φ. Solid lines correspond to y = 0.0, dotted lines to $\varphi = 0.025$ and dashed lines to $\varphi = 0.05$. Thin boundary layers can be observed near the hot and cold walls, while the isotherms in the core region become almost horizontal and parallel to adiabatic and impermeable walls. The temperature field reveals that the decrease of Da enhances heat transfer through cavity; the Da number is influencing the magnitude of the

Darcy term in the momentum eqn (12). With the increase of *Da* the flow regime is transited into the Darcy flow regime, which is close to the Darcy's law.

The addition of the nanoparticles into the base fluid seem to result in an attenuation of the convective motion inside the porous cavity. However, the overall heat transfer remains the same, since the main convective cell with upwards flows along the hot wall and downward flows along the cold wall is conserved.

The streamlines for $\phi = 0.4$ and $\varphi = 0.05$ are shown in Fig. 3. The flow field consists of single circulation flow in the clockwise direction as a result of the applied horizontal temperature difference. With a decrease of *Da*, the circulation becomes extended along the horizontal axis and the boundary layers become more significant.

The dependence of the solid volume fraction on the average heat transfer at different *Rap* and *Da* numbers is shown in Fig. 4. When conduction is a dominant heat transfer mechanism, $Da = 10^{-2}$, the heat transfer enhancement due to the increase of the solid volume fraction is more evident (Fig. 4a). The addition of nanoparticles into the base fluid enhances the thermal conductivity which results in higher values of Nusselt number. Moreover, the average *Nu* increases with increase of Ra_p. With decrease of $Da < 10^{-4}$, the *Nu* values no longer increase with φ (Fig. 4b and c). In that case, convection becomes the dominant heat transfer mechanism and is suppressed by the addition of nanoparticles into the base fluid and further enhancement of heat transfer rate is limited. When $Ra_p = 1{,}000$, it is obvious, that increase of φ leads to higher *Nu* when *Da* is high, while at low values of *Da* average *Nu* decreases along with the φ (Fig. 4d).

5 CONCLUSION

Natural convection phenomena of nanofluids in a porous media domain have been solved numerically using the BEM base numerical algorithm. A single phase mathematical model was used to describe the flow and heat transfer of nanofluids in porous media, while the Brinkman-Forchheimer momentum equation is used to describe the conservation of momentum. The influence of added *Cu*-nanoparticles into the base fluid on possible heat transfer enhancement was investigated, specially the volume fraction of nanoparticles, as well as different porous media properties.

In general, the addition of nanoparticles into the base fluid results in higher heat conductivity but it suppresses the natural convection phenomena. In case of non-Darcy regime, the heat transfer enhancement due to added nanoparticles is obvious which results in higher values of Nusselt number. However, in case of lower values of *Da,* when the model is approaching to the Darcy regime, the overall heat transfer is decreasing with the addition of nanoparticles.

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